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Infrared alignment of supersymmetric flavor structures

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The various experimental bounds on flavor-changing interactions severely restrict the low-energy flavor structures of soft supersymmetry breaking parameters. In this work, we show that with a particular assumption of Yukawa couplings, the fermion mass and sfermion soft mass matrices are simultaneously diagonalized by common mixing matrices and we then obtain an alignment solution for the flavor problems. The required condition is generated by renormalization group evolutions and achieved at low-energy scale independently of high-energy structures of couplings. In this case, the diagonal entries of the soft scalar mass matrices are determined by gaugino and Higgs boson soft masses. We also discuss possible realizations of this scenario and the characteristic sparticle spectrum in the models.

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I. INTRODUCTION

Since the standard model (SM) has been experimentally established below the weak scale, many kinds of attempts which extend the SM have been proposed so far. Among these, supersymmetry (SUSY) provides the most attractive candidate because of its various successful features of phenomenological applications such as the stability of mass hierarchy [1], the gauge coupling unification from the precise electroweak measurements [2], and so on. Moreover it has been found through intensive study that supersymmetric gauge theories have theoretically rich phase structures such as dualities and confinements. These properties have also been applied to try to construct more fundamental models beyond the SM.

Despite these attractive natures, the supersymmetric extensions of the SM generally possess a number of questions in addition to not being observed experimentally [3]. Some of these are caused by the existence of supersymmetric partners of the SM fields. They generally lead new contributions and sometimes disastrous effects, without some dynamical or symmetrical reasons, to the processes which should be suppressed. One of these questions is the flavor-changing neutral current (FCNC) problem. In the SM, the FCNC events are highly suppressed by the Glashow-Iliopoulos-Maiani (GIM) cancellation and its predictions are properly consistent with the experimental results. In the SUSY standard models, however, there are new sources of flavor-changing processes from the squarks and sleptons generation mixings [4,5]. These effects generally tend to overcome the successful SM predictions and then severely restrict the parameter forms involved.

To satisfy the FCNC requirements, several mechanisms have been proposed so far: the degeneracy of soft scalar masses [4], the decoupling of first two heavy generations [6], the alignment of quark and squark mass matrices [7], and so on. These mechanisms have been realized from various dynamics of high-energy (more fundamental) theories and the SUSY breaking effects; i.e., soft SUSY breaking parameters are thus given at some high-energy scale. On the other hand, since the FCNC processes are observed in the low-energy region, the renormalization-group (RG) evolutions between these two scales should be taken into account. These substantial effects have been used to obtain the approximate universal forms of SUSY breaking terms [8] and the heavy sfermion of the first two generations [9], where specific initial conditions of parameters at SUSY breaking scale are required to obtain sizable RG effects for avoiding the FCNC problems.

In this paper we show that with an assumption for Yukawa couplings, the flavor structure of squarks mass matrix is aligned with the corresponding Yukawa matrix below the SUSY breaking scale due to the RG running effects. This infrared alignment occurs provided that the low-energy values of Yukawa couplings are determined from the infrared fixed points of RG flow. Only with this simple and model-independent assumption is it found that the squark squared-mass matrix is diagonalized by the unitary matrices, which can also diagonalize the quark mass matrix (i.e., the fixed-point solutions), and then the FCNC processes are indeed suppressed. The manifestation of alignment is neither based on any special symmetries nor depends on detailed structures of high-energy theories. In this case, one may naively wonder whether small Yukawa couplings compared to gauge couplings can be realized as the RG fixed points. However, it can be actually accomplished, for example, by introducing heavy matter fields or by taking into account the contributions from extra spatial dimensions. We will discuss these realizations in more detail in a later section.

To suppress the FCNC processes to an acceptable level, a certain condition is generally required as in the RG-universality or RG-decoupling mechanisms stated above. In the present infrared alignment scenario, it is the rapid convergence of the fixed-point solutions. In the above examples

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of extra matter and extra dimensions, we actually obtain strong convergence into the fixed points. Moreover it has been known that the convergence of soft SUSY breaking parameters is expressed by the same factor as that of Yukawa coupling. Thus all properties of the mechanism are described by the Yukawa sector only and do not depend on detailed structures of SUSY breaking parameters at high-energy scale. In the following we will discuss these phenomena in addition to the characteristic pattern of sparticle spectrum predicted by the infrared alignment.

II. RG RUNNING OF SOFT TERMS

For later discussions, we first consider the fixed-point structures of soft breaking parameters in the SUSY standard models, and the convergence to the fixed points. As stated before, the convergence behavior is important to discuss the applications of fixed-point solutions to our low-energy physics. For this purpose we first work in the model with one gauge and one Yukawa couplings for simplicity. In realistic models they can be regarded as the $SU(3)_c$ gauge coupling and the top Yukawa coupling. It is straightforward to extend the obtained results to the case with more numbers of generation and the off-diagonal entries in soft SUSY breaking terms. We suppose the following superpotential:

$$W = yQuH_u, \tag{2.1}$$

where Q, u, and H_u denote quark doublet, quark singlet, and up-type Higgs superfield, respectively. The one-loop beta functions of the gauge coupling g and Yukawa coupling y are written as

$$\frac{dg}{dt} = \frac{1}{16\pi^2} bg^3, \tag{2.2}$$

$$\frac{dy}{dt} = \frac{1}{16\pi^2} y(a\bar{y}y - cg^2), \tag{2.3}$$

where $t = \ln \mu$ and the coefficients a, b, and c are $O(1)$ constants expressed in terms of group-theoretical indices. We also introduce the soft SUSY breaking terms, a gaugino mass, scalar soft masses, and a trilinear coupling

$$-L_{\rm soft} = \left(\frac{M}{2}\lambda\lambda + AQuH_u + \text{H.c.}\right) + Q^\dagger m_Q^2 Q + u^\dagger m_u^2 u + H_u^\dagger m_{H_u}^2 H_u, \tag{2.4}$$

where λ is a gaugino and Q, u, and H_u are the scalar components of corresponding chiral superfields. The beta functions for these soft SUSY breaking parameters are similarly obtained by the perturbative calculations of relevant one-loop Feynman diagrams. However, here we adopt a more convenient and useful way of calculations in the light of later discussions in the multigeneration case. It is known that the total Lagrangian with the soft SUSY breaking terms can be written in terms of superfields by introducing the Grassmannian parameter (external field) η [10]. With this formalism, the beta functions of SUSY breaking parameters can simply be written down by the so-called Grassmannian expansion method [11]. That is, the supersymmetric couplings g and y are replaced by

$$\tilde{g}^2 = g^2(1 + M\eta + \overline{M}\bar{\eta} + 2M\overline{M}\eta\bar{\eta}), \tag{2.5}$$

$$\tilde{y} = y - A\eta + \tfrac{1}{2} y(m_Q^2 + m_u^2 + m_{H_u}^2)\eta\bar{\eta}, \tag{2.6}$$

and are expanded by the Grassmannian parameter η. Then one obtains the beta functions of SUSY breaking parameters from the rigid SUSY ones [Eqs. (2.2) and (2.3)] as follows:

$$\frac{dM}{dt} = \frac{1}{16\pi^2} 2bg^2 M, \tag{2.7}$$

$$\frac{dA}{dt} = \frac{1}{16\pi^2}(3a\bar{y}yA - cg^2(A - 2My)), \tag{2.8}$$

$$\frac{d\Sigma}{dt} = \frac{2}{16\pi^2}(a(\bar{y}y\Sigma + \bar{A}A) - 2cg^2 M\overline{M}), \tag{2.9}$$

where Σ is a sum of the soft scalar masses; $\Sigma = m_Q^2 + m_u^2 + m_{H_u}^2$. (In the case with more Yukawa couplings, the sum Σ is defined corresponding to each Yukawa coupling.) From these beta functions, one can find the infrared stable fixed-point solutions:

$$\bar{y}y = \frac{b+c}{a} g^2, \quad A = -My, \quad \Sigma = M^2. \tag{2.10}$$

The fixed-point values are written in terms of the gauge coupling g and the gaugino mass M. The solutions for SUSY breaking parameters A and Σ can also be derived directly from the solution of Yukawa coupling y by use of the Grassmannian expansions. Here it is important to note that the fixed-point solutions for the soft SUSY breaking parameters do exist *only when* the corresponding Yukawa coupling has a stable fixed point. Therefore in the usual three generations case that the first two generations have tiny Yukawa couplings, the soft terms of these generations do not have stable infrared fixed points like Eq. (2.10).

Before proceeding to the generic three-generation case, we consider the convergence of couplings to the fixed-point solution Eq. (2.10). For this, let us analytically integrate the RG equations. In the present case, i.e., with only one Yukawa coupling, the results are expressed in a simple form as

$$X(t) = \xi X_0, \quad Y(t) = \xi Y_0, \quad Z(t) = \xi Z_0, \tag{2.11}$$

where $\xi \equiv (g(t)/g_0)^{2(1+c/b)}$ and the subscripts 0 imply they are initial values at some high-energy scale. The combined couplings X, Y, and Z are defined by

$$X = \frac{R-1}{R}, \tag{2.12}$$

$$Y=\frac{S-1}{R}-\left(1+\frac{c}{b}\right)\frac{R-1}{R}, \tag{2.13}$$

$$Z=\frac{T-1}{R}-\frac{(S-1)(\bar{S}-1)}{R}-\left(1+\frac{c}{b}\right)\frac{S-1}{R}$$
$$-\left(1+\frac{c}{b}\right)\frac{\bar{S}-1}{R}+\left(1+\frac{c}{b}\right)\left(\frac{c}{b}\right)\frac{R-1}{R}, \tag{2.14}$$

where

$$R=\frac{a\ \ \overline{y}y}{b+c\,g^2},\qquad S=\frac{-A}{My},\qquad T=\frac{\Sigma}{M\bar{M}}. \tag{2.15}$$

The fixed-point solutions Eq. (2.10) correspond to $X=Y=Z=0$ (in other words, $R=S=T=1$). It can be seen from the integrated forms [Eq. (2.11)] that the parameter ξ denotes the convergence factor, that is, the rate at which the couplings approach their fixed-point values. A sufficiently small value of ξ assures that the couplings converge to the infrared fixed points rapidly. As will be seen below, it depends on this smallness of ξ how the FCNC processes, which involve the off-diagonal elements of soft mass matrix, can be suppressed. Interestingly enough, all the RG fixed points including the soft SUSY breaking parameters are controlled by the single convergence factor ξ. Thus the FCNC problems of SUSY breaking parameters may be studied only from the rigid SUSY sector, and the initial values of these couplings are irrelevant to discussions. The factor ξ is expressed only by the gauge interaction part. In the four-dimensional theories, the smallness of ξ requires that the gauge couplings should be large at high-energy scale [12]. In this type of model, various phenomenological aspects, e.g., the Yukawa coupling textures, have been investigated [13].

III. INFRARED ALIGNMENT OF FLAVOR STRUCTURES

Now we further investigate the generation structures of soft breaking terms, from which one can discuss the flavor problems in supersymmetric theories. We consider the case where the RG evolutions are dominated by one gauge coupling g for simplicity. The superpotential takes the following form:

$$W=y^{ij}Q_iu_jH_u, \tag{3.1}$$

where i,j are generation indices, $i,j=1,2,3$. Here we consider the up-quark part only, but the results are easily applied to include the down-quark sector. The soft SUSY breaking parameters, m_Q^2, m_u^2, and A, are similarly extended to 3×3 complex-valued matrices. The scalar squared-mass matrices are Hermitian by definition. We now study the renormalization-group effects on the soft breaking parameters. A convenient way to see the behaviors of SUSY breaking terms under RG evolutions is the Grassmannian expansion method as we have used before. With this method, it is not necessarily needed to examine the RG equations explicitly. In the multigeneration case, the expansion of the rigid SUSY couplings are given by

$$\tilde{g}^2=g^2(1+M\eta+\bar{M}\bar{\eta}+2M\bar{M}\eta\bar{\eta}), \tag{3.2}$$

$$\tilde{y}^{ij}=y^{ij}-A^{ij}\eta+\tfrac{1}{2}[(m_Q^2y)^{ij}+(ym_u^{2\,\mathrm{T}})^{ij}+y^{ij}m_{H_u}^2]\eta\bar{\eta}. \tag{3.3}$$

Let us suppose that all the Yukawa couplings y^{ij} are determined by the infrared fixed-point values. This is the key assumption to obtaining an alignment of quark-squark flavor structures from RG evolution. The condition implies that the forms of the fixed-point solutions for SUSY breaking terms can be read from the above Grassmannian expansions. It should be noted that the expansion method can be applied only to the quantities concerned with the renormalization properties. We therefore apply it to the present situation that the couplings are determined from the anomalous dimensions of matter fields. Of course, the following argument of alignment does not hold for generic Yukawa solutions obtained from other mechanisms.

The fixed-point solution of Yukawa couplings may generally be given by $y^{ij}=c^{ij}f(g)$, or more exactly,

$$(yy^\dagger)^i_j=C^i_jF(g^2),\qquad (y^\dagger y)^i_j=\bar{C}^i_jF(g^2). \tag{3.4}$$

The function F does not carry generation indices because the gauge interaction is flavor blind. In the simplest case, F becomes $F\propto g^2$ but may have a more complicated form for other solutions such as quasifixed points.[1] The constant matrices C and $\bar{C}$ take different forms according to the properties of the doublet and singlet quarks Q and u. These matrices are Hermitian by definition and are diagonalized as

$$C'=V_L^\dagger C\,V_L=V_R^\dagger\bar{C}\,V_R=\text{diagonal} \tag{3.5}$$

with two unitary matrices V_L and V_R. The Yukawa coupling (the fixed-point solution c^{ij}) is also diagonalized by these matrices as $y'=V_L^\dagger y\,V_R=$ diagonal. Here, the prime means a matrix in the Yukawa-diagonal basis.

Let us perform the Grassmannian expansions about the fixed-point solution of $yy^\dagger$. From the expansion Eq. (3.3), we obtain

$$\widetilde{yy^\dagger}=yy^\dagger-Ay^\dagger\eta-yA^\dagger\bar{\eta}+[\tfrac{1}{2}y(y^\dagger m_Q^2+m_u^{2*}y^\dagger+y^\dagger m_{H_u}^2)$$
$$+\tfrac{1}{2}(m_Q^2y+ym_u^{2\,\mathrm{T}}+ym_{H_u}^2)y^\dagger+AA^\dagger]\eta\bar{\eta}. \tag{3.6}$$

Here and hereafter we omit the generation indices for simplicity. In the basis that the Yukawa matrix is diagonal, the solution can be rewritten as

[1]The quasifixed-point solutions may be obtained analytically even in multi-Yukawa coupling cases [14].

$$\widetilde{y'y'^{\dagger}} = y'y'^{\dagger} - A'y'^{\dagger}\eta - y'A'^{\dagger}\bar{\eta} + [\tfrac{1}{2}(y'y'^{\dagger}m_Q^{2\prime} + m_Q^{2\prime}y'y'^{\dagger}) + y'm_u^{2\prime}y'^{\dagger} + y'y'^{\dagger}m_{H_u}^2 + A'A'^{\dagger}]\eta\bar{\eta}, \tag{3.7}$$

where the redefined couplings A', $m_Q^{2\prime}$, and $m_u^{2\prime}$ are given by

$$A' = V_L^{\dagger} A V_R, \quad m_Q^{2\prime} = V_L^{\dagger} m_Q^2 V_L, \quad m_u^{2\prime} = V_R^{\dagger} m_u^{2\,\mathrm{T}} V_R. \tag{3.8}$$

These are nothing but the soft SUSY breaking parameters in the so-called super–Cabbibo-Kobayashi-Maskawa (CKM) basis (at the scale where the fixed points are realized). Therefore, if the off-diagonal elements of A', $m_Q^{2\prime}$, and $m_u^{2\prime}$ vanish, the supersymmetric FCNC processes are suppressed. With the expansions [Eqs. (3.2) and (3.7)], the linear term in η of Eq. (3.4) reads

$$A'y'^{\dagger} = -C'Mg^2\frac{dF}{dg^2}. \tag{3.9}$$

It is easily found that since the matrices y' and C' are diagonal, the off-diagonal entries in A' vanish. On the other hand, the diagonal elements, i.e., the eigenvalues of the matrix A are given by use of the leading term of Eq. (3.4),

$$A'^{ii} = -y'^{ii}M\frac{g^2}{F}\frac{dF}{dg^2}, \quad i=1,2,3. \tag{3.10}$$

Thus the trilinear coupling A is completely proportional to the Yukawa couplings on the fixed point. Again note that this proportionality is obtained from the assumption that the Yukawa couplings are determined by fixed-point values. This result is different from the infrared universality [15] in which the couplings are required to satisfy specific conditions by some symmetry and other theoretical assumptions, or there is no relation between the Yukawa and SUSY breaking couplings. In the present case, the coefficient matrices C and $\bar{C}$ do not necessarily satisfy any restricted conditions. The Yukawa couplings, trilinear terms, and soft scalar masses as seen below, take arbitrary forms in the Lagrangian but can be simultaneously diagonalized by the same mixing matrices. Note also that the CP phases of trilinear couplings are no longer free and fixed by that of gaugino mass and Yukawa couplings.

For the soft scalar masses, it is also easily found from the $\eta\bar{\eta}$ terms of Eq. (3.4) that the off-diagonal elements should satisfy

$$(m_Q^{2\prime})^i_j = (m_u^{2\prime})^i_j = 0, \quad i \neq j \tag{3.11}$$

provided that all the eigenvalues of the Yukawa matrix are not equal to each other; $|y'^{ii}|^2 \neq |y'^{jj}|^2$ $(i\neq j)$, e.g., if the mass hierarchy is properly realized. Here we have used the matrices y' and A' which are diagonal. In addition, the diagonal elements are given by

$$(m_Q^{2\prime})^i_i + (m_u^{2\prime})^i_i + m_{H_u}^2 = M^2\frac{d}{dg^2}\left(g^4\frac{d}{dg^2}\ln F\right), \quad i=1,2,3. \tag{3.12}$$

It is interesting to note that this expression does not completely depend on the model-dependent constants c^{ij}. In the end, we find that the supersymmetric FCNC problems are avoided on the fixed point. Namely, the quark and squark mass matrices are simultaneously diagonalized (''infrared alignment'') if the Yukawa couplings are determined by RG fixed-point solutions. Exactly speaking, the alignment suppresses the gaugino-mediated processes only. The higgsino-mediated processes still exist as a supersymmetric counterpart of the GIM-suppressed processes in the SM. These contributions are, however, small due to the weak couplings in the vertex.

As we have shown, the soft SUSY breaking parameters are not universal in the Lagrangian and may have rather generic structures. This is due to the remaining freedom of the mixing matrices V_L and V_R and also the fact that the soft mass eigenvalues are restricted only by several relations Eq. (3.12). In spite of these facts, interestingly enough, the FCNC problem is actually settled by quark-squark alignment. The mixing matrices, i.e., the solution c^{ij} is fixed in a model-dependent way but their detailed values do not affect the alignment of soft terms and moreover do not disturb the scalar mass spectrum either. It is clear from the above derivation that the alignment occurs in the sector where Yukawa couplings really converge to their fixed points. For example, when only eigenvalues are fixed, the flavor structures are not aligned between quarks and squarks although the relations like Eqs. (3.10) and (3.12) are obtained and somewhat restrict scalar mass matrix forms.

IV. ILLUSTRATIVE MODELS

Here we will comment on possible dynamics that may actually realize the infrared alignment mechanism. As argued above, the mechanism requires that Yukawa couplings, in particular even small ones, are reproduced as their fixed-point values. This situation naively cannot be accomplished in the usual supersymmetric standard models. That is, the RG evolutions produce only $O(1)$ fixed-point values of Yukawa couplings because the gauge couplings, which drive the Yukawa couplings, are also $O(1)$. This problem is, however, avoided in some extensions of the SM.

The first example is to add extra (vector-like) matter fields to the usual three-generation models. The mass scales of these extra matter fields are experimentally bounded from below and should be very large. With the heavy masses, the mixing Yukawa couplings between extra matter and our three-generation fields could be $O(1)$ and determined by fixed points of RG running above the heavy mass scales. Furthermore such heavy masses also depend on the fixed-point predictions. If these mass terms are generated from Yukawa couplings, the vacuum expectation values of relevant Higgs fields should be assumed as in the electroweak symmetry breaking. As an example, in case of three extra generations, the 6×6 mass matrix may take the following form:

$$\begin{pmatrix} & O(1)\,m \\ O(1)\,m & O(1)\,M \end{pmatrix}, \tag{4.1}$$

where m and M are light (electroweak) and heavy mass scales, respectively, and $O(1)$'s denote the fixed-point values of Yukawa couplings. In this case, the small Yukawa couplings of the first two generations are explained by the heavy mass suppressions like the seesaw mechanism and are realized as the fixed points. Thus the condition for alignment is satisfied and in the low-energy effective theory below M, the FCNC processes are suppressed.

Another important point to add extra matter fields is that, in general, it leads large gauge coupling constants at high energy. As a consequence, the convergence factor ξ defined in Eq. (2.11) becomes very small and the Yukawa and SUSY breaking couplings are driven into their fixed points very rapidly [16,17]. This strong convergence is surely required for the alignment to be achieved, i.e., to obtain enough suppressions of the off-diagonal elements of SUSY breaking parameters in the super-CKM basis. It is interesting that adding extra matter can provide two conditions required for the FCNC problem in one effort.

The second example we present is the models with extra spatial dimensions beyond the usual four dimensions. The quark and lepton chiral superfields as well as the gauge multiplets could be supposed to propagate through the extra dimensions. When the compactification scale M_c of extra dimensions is smaller than the cutoff Λ, the effects of RG evolutions between these two scales are enhanced by the contributions from numbers of Kaluza-Klein excited modes [18]. For instance, the beta function of Yukawa coupling y is given by

$$\frac{dy}{dt}=\frac{1}{16\pi^2}y\left[a\left(\frac{\Lambda}{\mu}\right)^{\delta_y}\overline{y}y-c\left(\frac{\Lambda}{\mu}\right)^{\delta_g}g^2\right], \tag{4.2}$$

where $O(1)$ coefficients a and c contain group-theoretical indices and the volume factors originated from the phase-space integral of Kaluza-Klein modes. Here we have neglected the logarithmic terms from the contributions of ordinary four-dimensional particles. The gauge beta functions are also written in a similar way. Compared to the four-dimensional case Eq. (2.3), the beta functions are amplified by the power factors $(\Lambda/\mu)^x\,(\geqslant 1)$. Roughly speaking, these powers just correspond to the number of Kaluza-Klein modes propagating in the loop diagrams. The integer exponents δ_g and δ_y are the largest gauge and Yukawa contributions, respectively, to the anomalous dimensions of matter fields. Of course, the four-dimensional results are recovered by taking the limit $\delta\to0$. These exponents are determined once we fix the configuration of relevant fields in the extra dimensions. Therefore δ_y and δ_g can generally take different values for various Yukawa couplings.

With the enhancement of RG evolutions, the suppressions of Yukawa couplings are established while the gauge couplings are order one. The hierarchy between generations can be generated by the difference of exponents δ_x. More interestingly, these suppressed Yukawa couplings are actually determined by the infrared fixed points and do not depend on high-energy input parameters. After all, it is found that the fermion-sfermion alignment occurs at the compactification scale M_c. In higher-dimensional models, two types of fixed-point scenarios can be realized by taking suitable field configurations in the extra dimensions: the quasifixed point [19] and the Pendleton-Ross type fixed point [20] scenarios. Especially in the case of Pendleton-Ross fixed points, even the smallness of CKM matrix elements may be explained and the supersymmetric FCNC processes are completely suppressed by the extra-dimensional mechanism.

Let us comment on the convergence factor ξ in this case, too. Since the couplings have power-running behaviors and the running region may be narrower than the four-dimensional case, one should also require the strong convergence in order to have enough FCNC suppressions. The rate of suppression ξ is now given by

$$\xi=\exp\left[\int_0^t dt'\frac{c}{8\pi^2}\left(\frac{\Lambda}{\mu}\right)^{\delta_g}g^2\right]\left(\frac{g^2(t)}{g^2(0)}\right). \tag{4.3}$$

This expression certainly reproduces the four-dimensional value (see Sec. II) in the limit $\delta_g\to0$. From this equation, we can see that there are two distinct ways to obtain a small value of ξ [20]. One is realized in asymptotically non-free gauge theories. This is clearly seen when the effect from the gauge anomalous dimension is dominant in the RG evolutions. The situation is almost similar to the ordinary four-dimensional cases [12]. The other possibility is essentially due to the existence of extra dimensions. That is, when the gauge contributions in the matter anomalous dimensions (the δ_g term) govern the RG equations, the suppression factor ξ becomes very small. This result holds even in asymptotically free gauge theories such as the minimal supersymmetric standard model. Then we may not necessarily require large gauge coupling constants at high energy unlike the four-dimensional cases and have many possibilities of model building.

V. SPARTICLE SPECTRUM

The present alignment realization leads to a rather characteristic scalar mass spectrum in the models. Here we will briefly discuss these spectra predicted by the eigenvalue relations of SUSY breaking parameters Eqs. (3.10) and (3.12). The relations give the boundary conditions of parameters in the low-energy effective theories at the scale at which the alignment occurs. It is noted that the relations also hold for the first and second generations. This is due to the fact that we have assumed that the Yukawa couplings of these generations are determined from the fixed points.

First, as noted before, the soft scalar mass terms are generally not universal forms but interestingly, the averaged squark masses are equal to each other and are given by the gluino and Higgs boson soft mass parameters. This is true for the first two generations even when the RG-running effects are included because of the small Yukawa couplings. On the

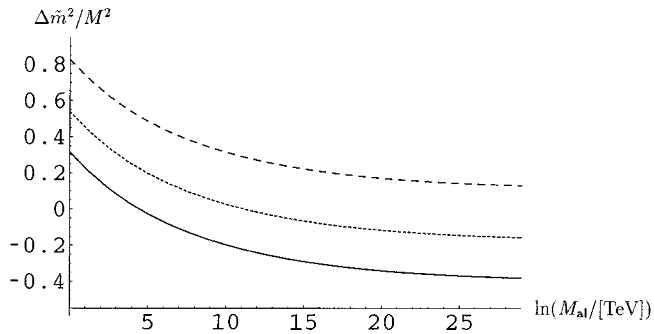

FIG. 1. Typical mass difference of the up and down squarks as a function of $M_{\rm al}$ at which scale the alignment occurs. The solid, dotted, and dashed lines correspond to $m_A=300$, 400, and 500 GeV, respectively. We take $\tan\beta=3$ and the gluino mass $M=500$ GeV at 1 TeV.

other hand, the averaged mass of the third generation may differ from the others due to the large top (and possibly bottom) Yukawa effects. Moreover for the third generation, there could be a large splitting between two mass eigenvalues by a large contribution from the trilinear coupling. For the first two generations, the A parameters are small [see the boundary condition Eq. (3.10)].

The results of Sec. III are also applied to the down-quark part and we here assume that the alignment takes place in the down sector. In this case, the soft masses relations such as Eq. (3.12) lead to more interesting features when taking into account the electroweak symmetry breaking. For example, minimizing the Higgs potential requires the condition, $m_{H_d}^2-m_{H_u}^2=-(m_A^2+M_Z^2)\cos 2\beta$, where m_A and M_Z are the masses of the neutral pseudoscalar Higgs and Z boson. Together with the mass relations, the symmetry breaking condition is written as

$$\tilde{m}_u^2-\tilde{m}_d^2=-(m_A^2+M_Z^2)\cos 2\beta, \tag{5.1}$$

where $\tilde{m}_x^2$ denotes the averaged squark masses. The right-hand side is positive since the condition $\cos 2\beta>0$ is inconsistent with the observed quark mass patterns. We thus find that the up-type squarks are necessarily heavier than the down-type ones. This characteristic spectrum is different from the usual cases, e.g., with the universal soft masses generated in the supergravity models. In that case, the up-type squarks, particularly in the third generation, become lighter than the down squarks at the electroweak scale because of the RG effects induced by large Yukawa couplings.

In the above we have considered that the alignment occurs at the electroweak scale. However, the conditions may generally be achieved at some higher scale. In the realistic models discussed in Sec. IV, this scale corresponds to the heavy mass scale of extra matter fields or the compactification scale of extra dimensions. For definiteness, we suppose that below these scales we have the minimal supersymmetric standard model. Then the difference of the low-energy stop and sbottom averaged masses $\Delta\tilde{m}^2\,(=\tilde{m}_t^2-\tilde{m}_b^2)$ is evaluated once one fixes the value of $\tan\beta$. In Fig. 1, we show a typical behavior of the mass difference $\Delta\tilde{m}^2$ as a function of $M_{\rm al}$ at which scale the alignment occurs, i.e., the soft masses relations are imposed. Here we take $\tan\beta=3$ and the gluino mass $M=500$ GeV at 1 TeV, but the mass difference behavior is almost independent of M. It is found from this figure that the stop mass can be smaller than that of sbottom as the scale of new physics $M_{\rm al}$ increases. In other words, if the squark masses are measured in the future, the new physics scale could be determined. As for the first and second generations, the up-type squarks are always heavier than the down-type ones even below the scale $M_{\rm al}$. Generally, the detailed information above the threshold $M_{\rm al}$ is lost, such as in the present models using infrared fixed points. However, the RG invariant relations among the soft SUSY breaking parameters [17] may be helpful in speculating on the SUSY breaking mechanism at high-energy scale.

We also comment on the masses of neutralino as a possible candidate of the lightest SUSY particle (LSP) in this scenario. As above, if we take into account the electroweak symmetry breaking properly, the supersymmetric Higgs mass parameter μ is written as $\mu^2\simeq -m_{H_u}^2-1/2M_Z^2$ for not too small a value of $\tan\beta$. Suppose that the alignment is realized at high energy and then the soft mass relations are imposed at that scale, the low-energy value of μ parameter is given by

$$\mu^2\simeq\tilde{m}_t^2-xM^2-\tfrac{1}{2}M_Z^2, \tag{5.2}$$

where the coefficient x denotes the RG-running effect $[x(0)=1]$. It takes the value 0.8–1.0 depending on the alignment scale, but is almost independent of $\tan\beta$ and the initial value of M. The averaged mass $\tilde{m}_t^2$ can vary with different initial ratio of two soft scalar masses because only the sum of them is known at the boundary. The value of the μ parameter roughly gives the Higgsino-like neutralino mass. Comparing Eq. (5.2) with the bino mass, we find that may be a candidate for the LSP. If one assume the universal gaugino masses at some high-energy scale, the bino mass becomes lighter than the gluino and then it is found from Eq. (5.2) that the LSP is bino in almost all parameter space. However more involved boundary condition of gaugino masses could naturally induce the Higgsino-like neutralino LSP. Of course this naive analysis does not include the lepton sector, gravitino, etc., and clearly more exact treatment should be performed, for example, by embedding the models in grand unified frameworks.

Finally, the FCNC problems in the case of such a higher-scale alignment should be discussed. As for the gluino-mediated FCNC processes, it is clear that they are completely suppressed even in the low-energy region because of the alignment property. This is different from the case of universal soft SUSY breaking terms. In that case, the Yukawa matrices are not diagonal in general. Then the off-diagonal elements of the soft mass-squared matrices are induced under the RG evolutions and would lead the FCNC problems. On the other hand, in the alignment mechanism, the Yukawa and soft mass matrices are simultaneously diagonalized and then the radiative corrections do not cause the FCNC processes at low energy. However the charginomediated diagrams still exist even in the present case. Although these processes are suppressed by small coupling constants, they would lead some bounds for degeneracy between the diagonal elements of the soft mass matrix of left-handed-type squarks. If that is the case, the soft mass relations for the first two generations could restrict the scalar spectra in the models. Notice that one may easily avoid these bounds by arising the scalar masses (and gaugino masses due to the boundary conditions from the alignment). A detailed analyses of sparticle mass patterns including the LSP and also other problems, e.g., the charge and color breaking minima involving generation mixing couplings, will be discussed elsewhere.

VI. SUMMARY

In this paper we have shown that the quark mass and the squark squared-mass matrices are diagonalized simultaneously in the case where Yukawa couplings (quark masses) are determined from the anomalous dimensions, i.e., the fixed-point solutions of RG equations. This result provides a natural alignment solution of the supersymmetric FCNC problems. On the fixed point, the soft SUSY breaking parameters are not universal but can take arbitrary forms in the Lagrangian. This is because: (i) the soft mass eigenvalues are determined only from several relations and (ii) the flavor structure of Yukawa couplings, i.e., the diagonalizing matrices of the Yukawa matrix, are not fully fixed by observation. In spite of these unconstrained mass patterns, the FCNC processes are actually suppressed at low energy.

We have also discussed several possible dynamics realizing the infrared alignment. As examples, we have presented the models with extra heavy fields or with extra spatial dimensions beyond the SM. One of the important points for applying the fixed-point solution to the FCNC problem is that the SUSY breaking parameters actually converge to their fixed-point values at an accurate level. We have shown that this strong convergence can be naturally obtained in these models.

Furthermore the present alignment also restricts the eigenvalues of soft SUSY breaking couplings even in the first two generations. The scalar trilinear couplings are proportional to the corresponding Yukawa couplings and cannot have new CP-violating phases. In addition, the eigenvalues of squark squared-mass matrices for all generations are determined by gaugino masses from sum rules. These results give a peculiar pattern of sparticle mass spectrum. For example, the up-type squarks are generally heavier than the down-type squarks. Considering the mass relations as boundary conditions at some high-energy scale, we also discuss the typical RG-running effects below that scale and possible candidates of the lightest SUSY particle. The sparticle spectra may become rather different from those in other mechanisms for suppressing the FCNC processes and would give distinctive signatures in future experiments.

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